# Validation of Models of Plates with Discontinuities Made of Plastic Materials, through Modal Analysis

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The paper presents an experimental validation, through modal analysis, of some models of plates with discontinuities, made of plastic materials. The numerical study was undertaken using the Finite Element Method. The geometrical model was obtained with nondestructive evaluations using computer tomography, allowing thus to find the shape, position and dimensions of defects. The good agreement between the experimental and numerical results led to the validation of the investigated models.

Key-words: defect, modal analysis, computer tomography, numerical analysis

The dynamic behavior of structures is very important for the understanding and evaluation of their performances

Appearance of discontinuities (defects) in different materials can lead to the modification of eigenfrequencies, yielding thus to malfunctioning of structures and even to their failure [2].

Dynamic experimental investigations can be used in order to emphasize the influence of defects on the behavior of structures. The most used method is the modal analysis, through which a study of vibrations eigenmodes can be undertaken [1, 3]. The obtained results can establish the existence of defects in a structure, but without offering information on the characteristics of discontinuities.

In this paper, a dynamic study is presented for plates made of plastic materials having internal defects. Both experimental modal analysis and numerical analysis using the Finite Element Method (FEM) are considered in order to validate the models of the investigated plates.

An initial non-destructive investigation prior to the numerical calculations is necessary in order to determine the location, shape and dimensions of the defects. X-ray Computer Tomography (CT), [4], was used for this purpose.

Achievement of the geometric model using X-ray Computer Tomography

In order to obtain the position, shape and dimensions of the defects, X-ray Computer Tomography was used [4-6].

The device used to investigate the plates was a spiral computer tomograph – helical CT scan. This has as the main feature the very fast acquisition of high quality images due to the improvement of the technology of the detector. The used spiral CT, named also multi-detector computer tomograph, was a four section type system.

The principle of computer tomography is based on:

-measurement of attenuation of a narrow X-ray fascicle that travels with a circular movement through an width equivalent layer in the examined structure;

-reconstruction of the image from its different projections [6].

Using specialized software, the images obtained through CT scan of the investigated plates were processed with a dedicated methodology, according to which two planes of interest (vertical and horizontal) were defined in the cross section where the opening of the defect is maximum.

Information obtained from the processing of the plates made of polyester resin are presented in figure 1 (for

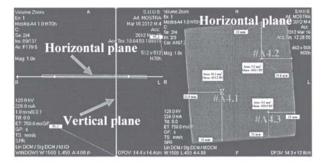


Fig. 1 Processing of CT scans – specimen A<sub>4</sub>

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Defect	Size of defect	Position of defect		
Defect	[mm]	Coord. X [mm]	Coord. Y [mm]	depth [mm]
#A4.1	Ф 3.5	31.9	42	1.3
#A4.2	Ф 2.1	70.2	68	1.3
#A4.3	Φ1	75	21	1.3

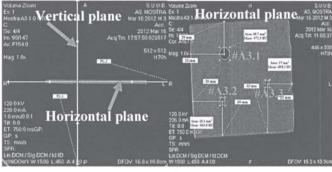
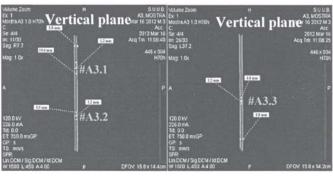


Fig. 2. Processing of CT scans – specimen A<sub>3</sub>



Size of defect Position of defect Defect Coord, X [mm] [mm] Coord. Y [mm] depth [mm] #A3.1 Ф 10.6 33 1.2 #A3.2 Ф 5.9 30 #A3.3 Ф 4.6 43

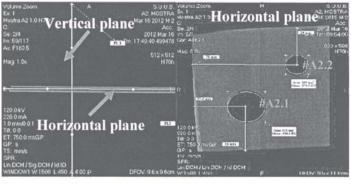


Fig. 3. Processing of CT scans – specimen A<sub>2</sub>



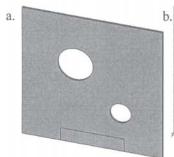
specimen  $A_4$ ), in figure 2 (for specimen  $A_3$ ) and in figure 3 (for specimen  $A_2$ ). Tables 1-3 shows all the obtained results about the size and position of defects found using CT. The

same co-ordinate system was used for all considered plates

Defect	Size of defect	Position of defect		
Defect	[mm]	Coord. X [mm]	Coord. Y [mm]	depth [mm]
#A2.1	Ф 25.7	37.4	38	1.3
#A2.2	Ф 15.5	70	70	1.3

**Table 3**CHARACTERIZATION OF DEFECTS – SPECIMEN A<sub>2</sub>





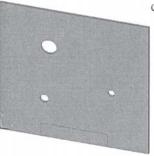




Fig. 4. The calculus model

Fig. 5. The geometry of the middle layer: a. Plate A<sub>2</sub>; b. Plate A<sub>3</sub>; c. Plate A<sub>4</sub>

Material	Young's modulus [N/m²]	Poisson's ratio	Density [kg/m <sup>3</sup> ]
Polyesteric resin	3.48·10 <sup>9</sup>	0.28	1200

Table 4
CHARACTERISTICS OF THE
MATERIAL

**Table 5**THE FIRST EIGENFREQUENCY OF PLATES

Plate	$A_4$	$A_3$	$A_2$
Frequency [Hz]	86.33	83.98	85.94

Using the obtained information, the geometrical models of the plates were elaborated in order to determine the eigenfrequencies and eigenmodes with FEM [7-8].

# The modal numerical analysis

The calculus models (fig. 4) were the same for all specimens, except for the geometry of the middle layer, which is presented in figure 5 for each plate.

The geometric models were meshed using tetrahedral elements that allow modeling irregular shapes without losing precision of the analysis [8]. Taking into account that, in the experiment, the plates were clamped using a jaw vice, an elastic fixed end was modeled in the finite element analysis using spring elements. The characteristics of the used material are presented in table 4.

The values of the first eigenfrequency obtained through numerical analysis are listed in table 5.

The shape of the first eigenmode is the same for each plate and it is shown in figure 6.

#### Experimental modal analysis

The modal analysis allows the user to achieve a modal model which is suitable for investigation of structural modifications [1, 3]. Three stratified plates with imposed defects as presented above, made of three layers of plastic materials (polyesteric resin), were considered in the



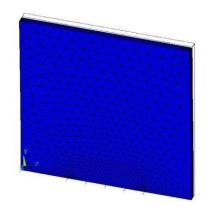


Fig. 6. Shape of the first eigenmode

experimental analysis. The specimens are square-shaped with a size of 100 mm. The thickness of each layer is 1.3 mm and the defects are located in the middle layer.

The experimental set-up consists in a Pulse system, a piezoelectric accelerometer and a modal hammer – Brüel & Kjaer (fig. 7).

Each plate was initially clamped in a jaw vice on one side and excited using the 8206 B&K modal hammer with steel head. A 4514 B&K piezoelectric accelerometer was used to measure the response (fig. 8).

The experimental data was analyzed with the dynamic analysis software of the Pulse system, obtaining thus the frequency response curves in the form of mobility (ratio between velocity and force).

The experimental values of the first eigenfrequency for each plate are listed in table 6.







Fig. 7 The experimental set-up: a. Pulse system; b. Piezoelectric accelerometer; c. Modal hammer

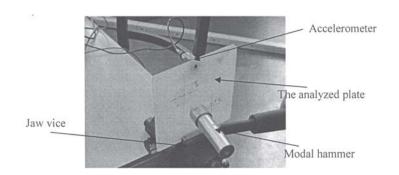


Fig. 8 Modal analysis of the plates

Townson.	Plate	Dimensions [mm]	Frequency [Hz]
-	A .	Difference [iffin]	20
	A A	100 - 100 - 2.0	75
	A <sub>3</sub>	$100 \times 100 \times 3.9$	/3
-	$A_2$		78

**Table 6** FIRST EIGENFREQUENCY OF PLATES

	Case 1	Φ12.5 (50,50)	Φ25 (50,50)	Φ50 (50,50)
Frequency [Hz]		87	88.1	88.7
	Case 2	Ф25 (50,30)	Ф25 (50,60)	Φ25 (50,87)
Frequency [Hz]		87	88	89

 Table 7

 RESULTS OF THE NUMERICAL SIMULATIONS

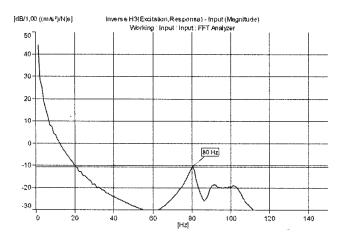


Fig. 9 Mobility curve for plate A<sub>4</sub>

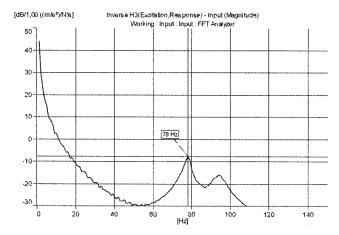
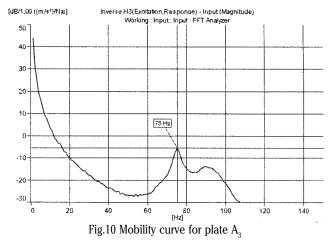


Fig. 11 Mobility curve for plate A,



The frequency response curves in the form of mobility are shown for each plate in figures 9-11.

**Comparative analyses** 

For each plate, the first experimental eigenfrequency was compared to the one obtained using the finite element method. Due to the small dimensions of the plates, meaning an elevated stiffness, only the first eigenfrequency was

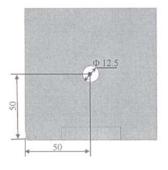
determined experimentally. The other eigenfrequency were too small and they could not be separated from the signal noise of the device.

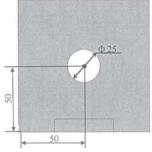
The numerical results were validated by the experimental analysis for the plate with defects made of plastic materials, since the errors were less than 10%. Analysis of the results reveal the modification of the dynamic properties with the changes of the structural integrity of the studied structures, justifying thus the necessity of this study.

A good correlation of the percentage of discontinuities with the value of the first eigenfrequency was noticed. The nonlinearity of the phenomenon of interdependence between the percentage of discontinuities and the first eigenfrequency can be explained by the fact that the eigenfrequencies have a nonlinear dependence with the mass and stiffness of the structure. The position of the discontinuities have also a great importance in the influence on the material defects on the dynamic behavior.

### **Numerical simulations**

Following the validation of the model, some numerical simulations were undertaken in order to observe the influence of the structural modifications on the dynamic behavior. The considered models respect the same working parameters as the experimentally validated





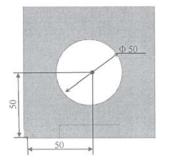
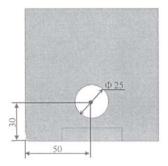
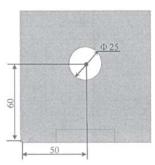


Fig. 12 Case 1: The size of the defect is variable and its position is constant





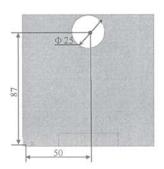


Fig. 13 Case 2: The position of the defect is variable and its size is constant

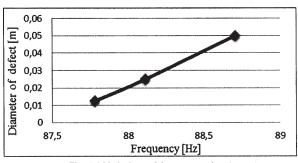


Fig. 14 Variation of frequency for Case 1

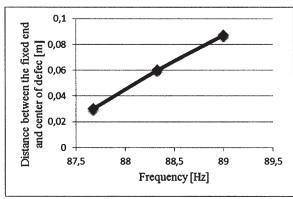


Fig. 15 Variation of frequency for Case 2

calculus model (dimensions, material, constraints, type of finite element, etc.). A single defect, positioned in the middle layer was considered.

Two possibilities were taken into account:

-modification of the defect diameter, keeping its position (fig. 12);

-modification of the position of the defect, keeping its diameter (fig. 13).

The values of the first eigenfrequency for each case are listed in table 7.

The variation of the frequency for the two analyzed situations are presented in graphical form in figures 14 and 15.

An increase of the first eigenfrequency is observed as a result of the increase of both the dimensions and the distance from the fixed end to the center of defect.

# **Conclusions**

The modal analysis undertaken in this paper both experimentally and numerically lead to the validation of the numerical models of plates made of plastic materials and having inner defects.

The models were obtained using X-ray computer tomography, through which a complete, precise and detailed evaluation of the structure can be performed. The dimensions of the structures and the shape, position and dimensions of defects can be obtained using this method. The collected data can be used for building correct numerical models, further analyzed by the finite element method.

The validation of the numerical models was the main goal of this study since an analytical approach is extremely difficult or even impossible.

The main conclusion of the present numerical studies is that the defects in the material can influence the dynamic behavior of structures, such influence depending on many factors as: volume percentage of the discontinuities, their shape and position, etc.

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